Packet Networks Modelling by BCMP theorem

by

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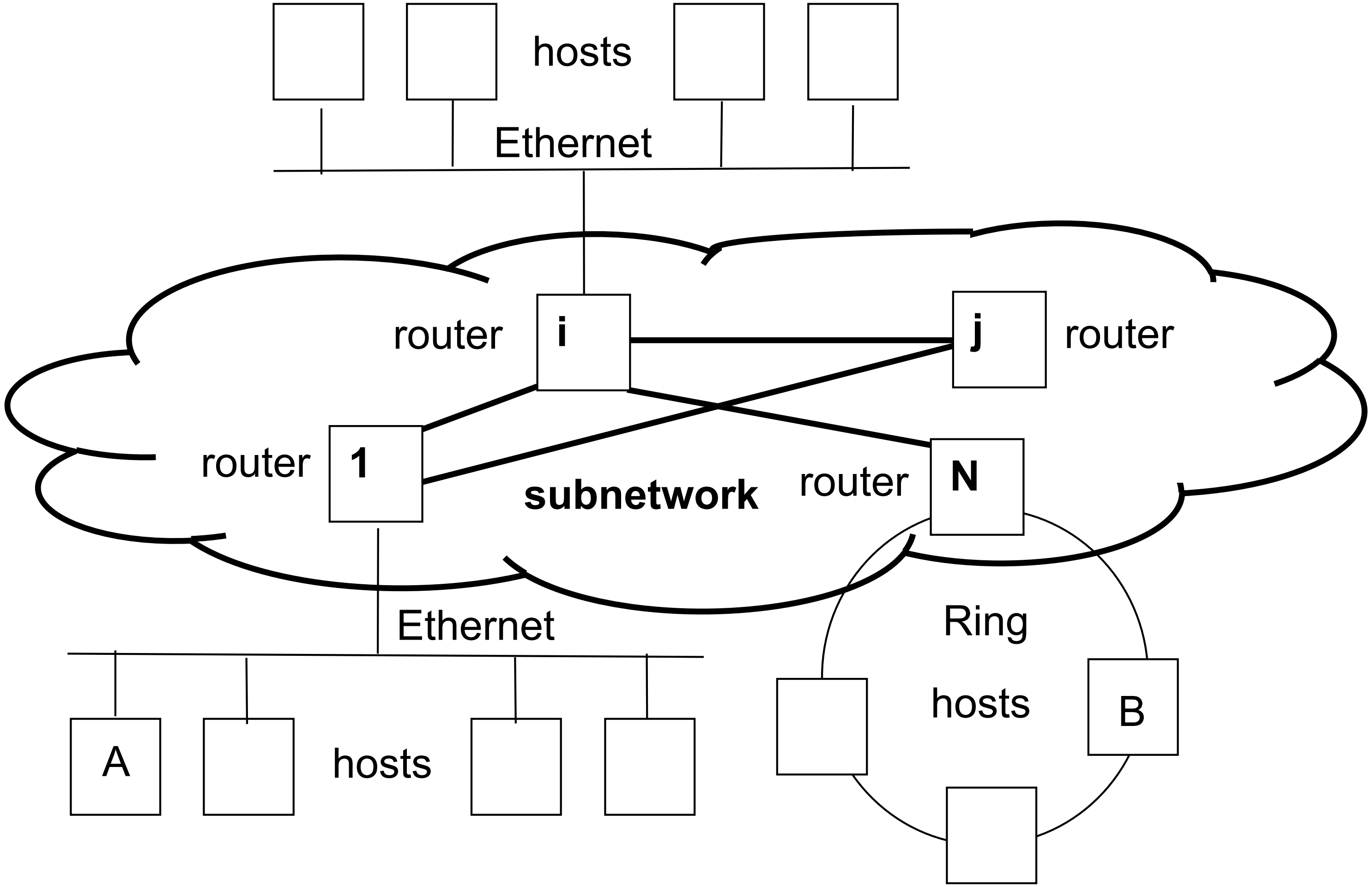
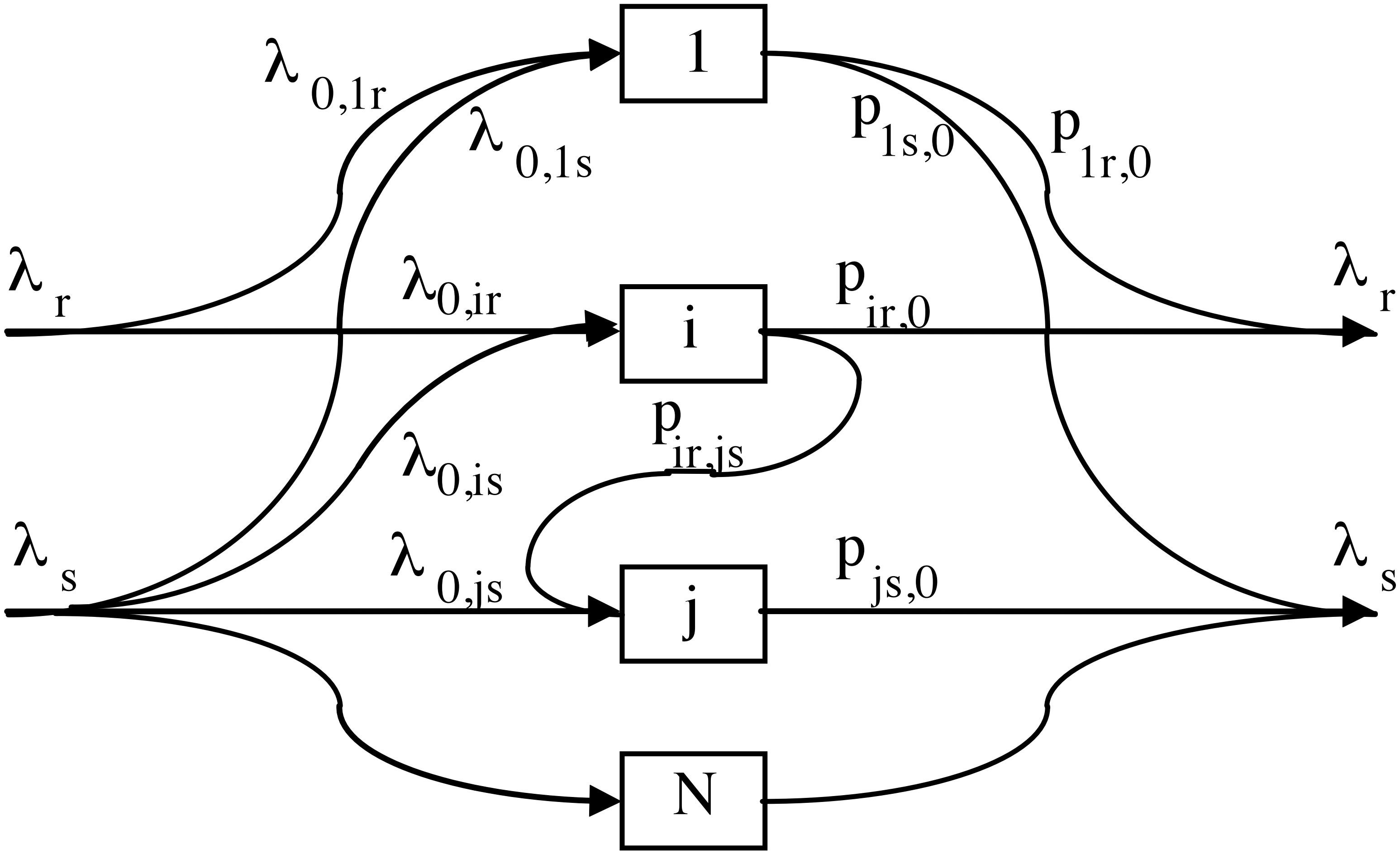
Internet Subnetwork



The Model: BCMP Open Network



Average Input Rates Per Node

$$\lambda_{js} = \lambda_{0,js} + \sum_{i,r} \lambda_{ir} \cdot p_{ir,js}$$ for r, s = 1,2,3,4. (1)

$$\lambda_{j} = \lambda_{0,j} + \sum_{i} \lambda_{i} \cdot p_{i,j}$$ per class (3)

or in matrix form:

$$\left[\lambda_1 \lambda_2 \ldots. \lambda_N\right] = \left[\lambda_{01} \lambda_{02} \ldots. \lambda_{0N}\right] + \left[\lambda_1 \lambda_2 \ldots. \lambda_N\right] \times \begin{bmatrix} p_{11} p_{12} \cdots p_{1N} \\ p_{21} p_{22} \cdots p_{2N} \\ . \\ . \\ p_{N1} p_{N2} \cdot p_{NN} \end{bmatrix} \qquad (4)$$

Solution

$$\|\lambda\| = \|\lambda_0\| \times \|p\|^{-1} \; where$$

$$\|p\| \equiv \begin{bmatrix} 1 - p_{12} - p_{13} \cdots - p_{1N} \\ - p_{21} 1 - p_{23} \cdots - p_{2N} \\ \cdots \\ \cdots \\ - p_{N1} - p_{N2} \cdots\cdots\cdots 1 \end{bmatrix} \quad (5)$$

Example

For p =

$$\begin{matrix} 1.0000 & -0.1000 & -0.2000 & -0.3000 \\ -0.1000 & 1.0000 & -0.2000 & -0.3000 \\ -0.1000 & -0.2000 & 1.0000 & -0.3000 \\ -0.1000 & -0.2000 & -0.3000 & 1.0000 \end{matrix}$$

$$\|\lambda\| = \|\lambda_0\| \times \begin{Vmatrix} 1.1364 & 0.3220 & 0.4647 & 0.5769 \\ 0.2273 & 1.2311 & 0.4647 & 0.5769 \\ 0.2273 & 0.3977 & 1.2981 & 0.5769 \\ 0.2273 & 0.3977 & 0.5288 & 1.3462 \end{Vmatrix}$$

Knowing λ_{ir} and T_{sr} for node i and class r follows $\rho_{ir} = \lambda_{ir} * T_{sr}$ that is the node (router) i utilization for class r. Summing over all classes we get an aggregate utilization for node i as $\rho_i = \Sigma_r \rho_{ir}$.

Sojourn and Total Delay

Sojourn time CDF for class r:

$$S(y) = 1 - e^{-\mu_r (1-\rho) y}$$

Sojourn time pdf Laplace transform for class r in node i:

$$L_i(s) = \frac{\mu_r (1 - \rho_i)}{\mu_r (1 - \rho_i) + s}$$

Total delay Δ pdf Laplace transform:

$$L_\Delta(s) = \prod_i \frac{\mu_{ri} (1 - \rho_i)}{\mu_{ri} (1 - \rho_i) + s}$$

Total delay pdf for class r:

$$S_\Delta(y) = \sum_i \frac{c_i \left(1 - e^{-\mu_{ri} (1-\rho_i)}\right)}{\mu_{ri} (1 - \rho_i)} \qquad \textbf{(13)}$$

Jitter Distribution

Average total delay:

$$\Delta_s = \sum_i \frac{1}{\mu_{ri}(1-\rho_i)} \tag{14}$$

Jitter distribution:

$$\begin{aligned} & P[|g| \le y] \\ &= P[\Delta_s - y < \tau \le y + \Delta_s] \\ &= P[\tau \le \Delta_s + y] - P[\tau \le \Delta_s - y] \\ &= S_\Delta(\Delta_s + y) - S_\Delta(\Delta_s - y) \end{aligned} \tag{15}$$

Delay Calculations Between Hosts A and B

Router utilizations: ρ_1 = 90%, ρ_i = 60% and ρ_N = 80%.

Average voice packet service times: $T_{sr} = 1/\mu_r$ = 10 msec

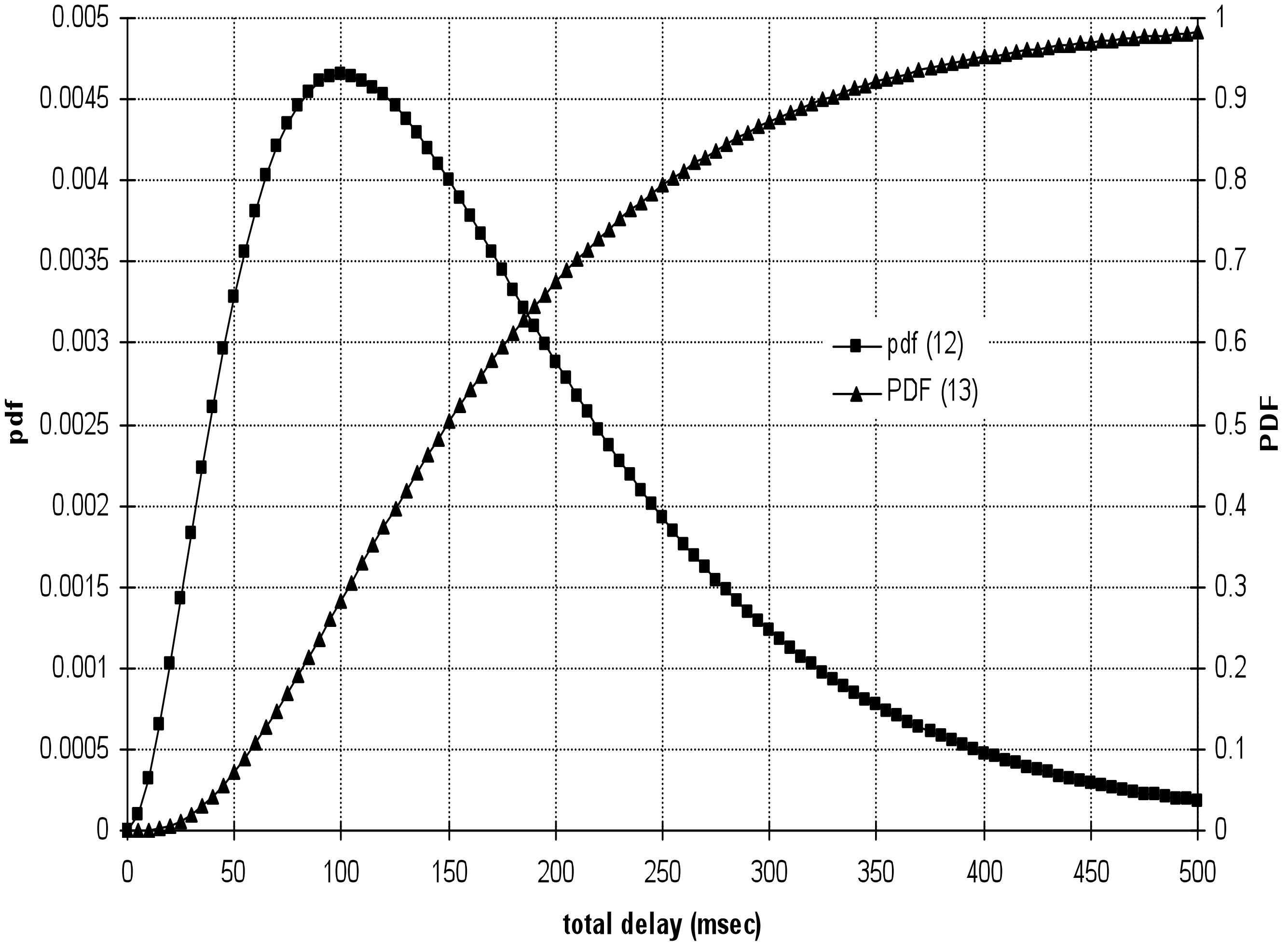
Total delay Δ:

average = 10/0.1 + 10/0.4 + 10/0.2 = 175 ms (14)

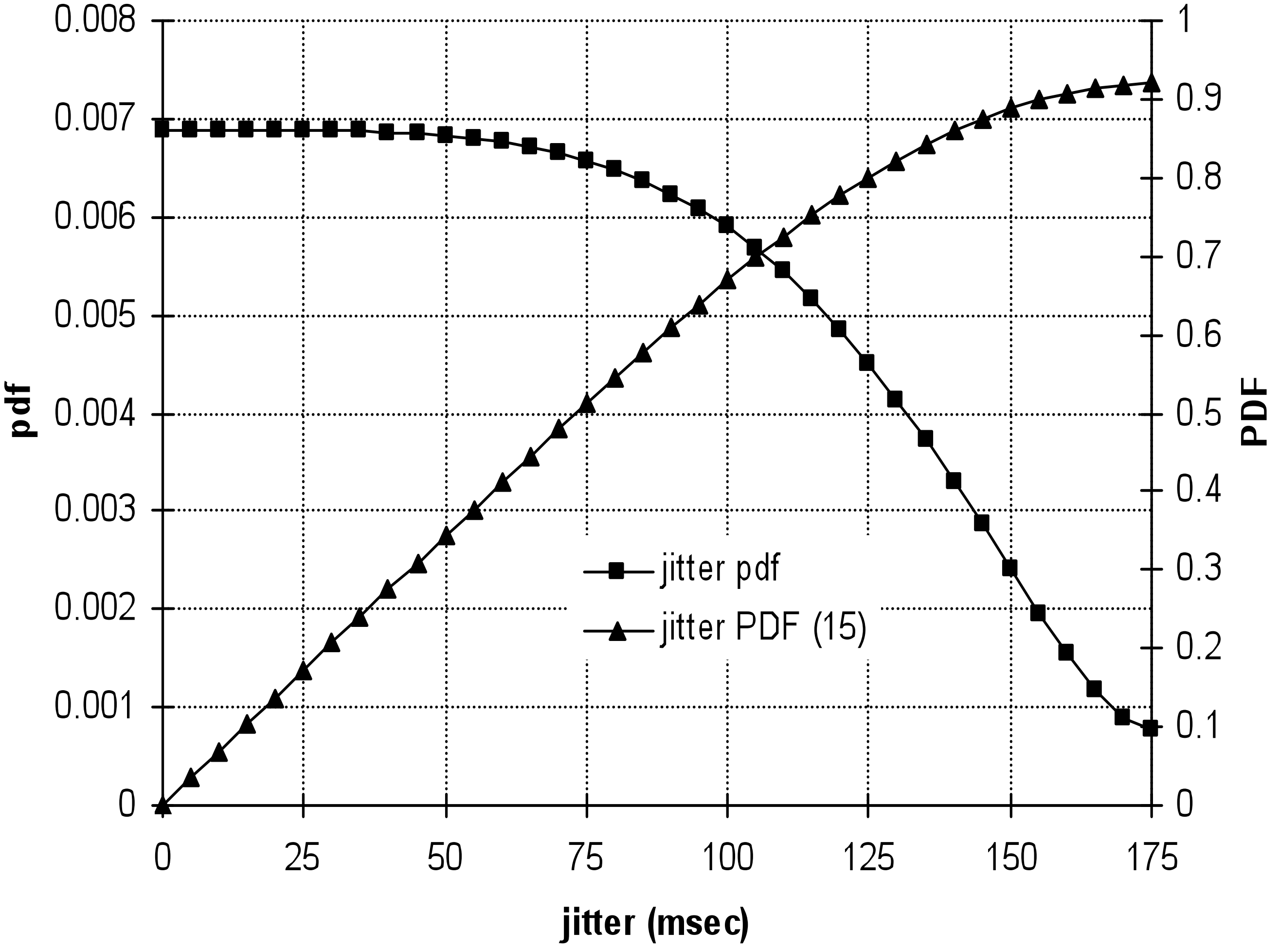
pdf: $s_\Delta(y)$ = 0.0267 exp(-0.01y) + 0.0133 exp(-0.04y) – 0.04 exp(0.02y) (12)

CDF: $S_\Delta(y)$ = 1 – 2.67exp(-0.01y) – 0.33exp(-0.04y) + 2exp(-0.02y) (15)

Delay Distributions Between Hosts A and B



Jitter Distributions Between Hosts A and B



Conclusion

- We have shown how to use the BCMP theorem to model Internet (packet) type of networks for any class of packets and processor sharing type of node to avoid bottlenecks.
- Under assumptions of M/M/1 sojourn time per node we gave closed form solutions for total delay and jitter distributions between any two nodes.